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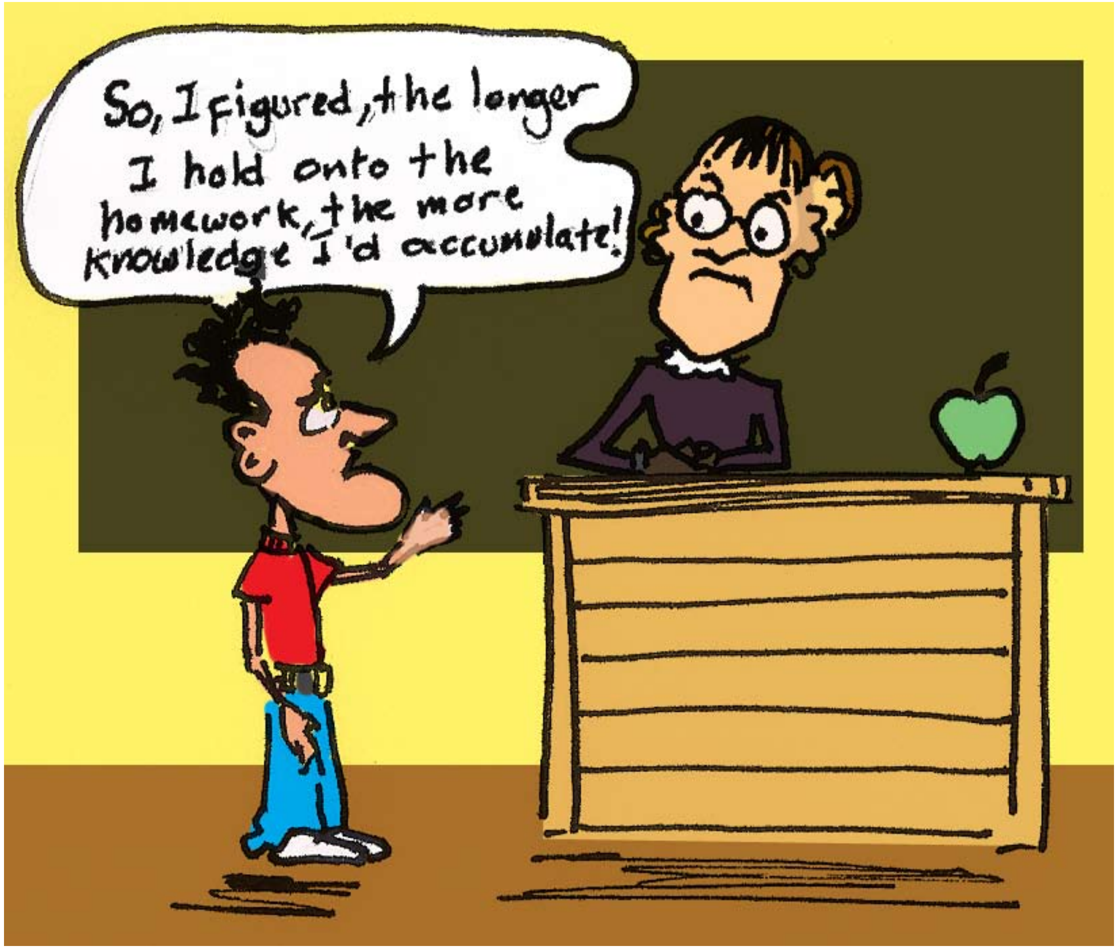
When Harry Met Kelly

Had such a meeting of minds occurred would the horse have ended up before the cart?



Kelly's School Days

In my last *Wilmott* column, "The Education of a Quant", I considered what might have happened if Harry Markowitz had known John Kelly. Instead of deciding that investors diversified to reduce risk, he might have theorized that investors diversified in order to get an optimal level of risk. The history of finance might have taken an interesting path.

Markowitz's actual creation, Modern Portfolio Theory (MPT), holds that investors care about the statistical properties of their portfolios and act to maximize expected return subject to a risk constraint. Expected return is good, risk is bad, and the investor balances the two.

A Kelly-inspired analogue might have been called Investment Growth Theory (IGT). The claim would be that investors selected portfolios for optimal long-term growth. Investors care about investments, not portfolios, and determine how much to allocate to each based on expected return and variance. There are portfolio effects — your capital allocation decision on each investment depends on your past decisions — but these are second-order.

MPT is top-down. An investor selects parameters based on risk preferences and an algorithm selects the portfolio that best satisfies them. IGT is bottom-up. An investor considers investments, makes an allocation decision, then moves on to the next decision. The portfolio is what results from this process.

IGT is clearly a better description of the world. No investors used top-down approaches when Markowitz wrote. People have tried it since, inspired by what MPT said they should do, but it has never been popular or conspicuously successful. When it is used, it is generally used only at the asset-class level rather than to select individual positions; and it is constrained tightly to force a result similar to preconceived ideas.

IGT also seems to be a better description of investor thought processes. Investors focused on how much capital was at risk in a position, what the expected return was, and how much variance of return could be expected. These are the three most important parameters in Kelly investing. MPT suggested the most important question was correlation among investments and that standard deviation mattered, not variance. Although the two might seem to be the same, as standard deviation is the square root of variance, they differ in one key respect. In order to compare standard deviation to expected return, you have to specify a time horizon; while variance and expected return have the same ratio at any time horizon. MPT was a one-period theory: you picked the best portfolio then at the end of the period, you traded it in for the portfolio optimized for the next period. IGT is a dynamic theory without a fixed time horizon, in fact, specifying a horizon undermines the assumptions of the theory.

Another virtue of IGT is that it can handle short positions and derivatives naturally. MPT requires some kind of investment constraint (such as total dollars spent) and these can be tricky to define once you get away from long-only asset positions. In fact, in the IGT world, there is no difference between investors and issuers of securities. MPT assumes security issuance and terms are exogenous. It doesn't try to explain them; it tells investors how best to react to them.

Both theories had trouble explaining the make-up of typical portfolios. MPT argued for much more diversification than was common in the 1950s and 1960s, while IGT, at least in naïve application, recommended much more risk.

MPT's big advantage is it corresponded better to what investors and managers thought they were doing. They might proceed bottom-up and look mostly for investments with better-than-average expected returns rather than investments with the right correlations, and they might have no idea what their portfolio standard deviation or time horizon was; but judging them on the ratio of return to standard deviation seemed reasonable. A manager in the 1950s might have agreed that MPT was a decent simplified model of portfolio construction, and Markowitz did market it as a product with some limited success. No one thought they were IGT investors, and until Ed Thorp, no one tried to market an IGT product.

The next advance in finance was the Efficient Market Hypothesis (EMH). This held that all securities were priced fairly. In the MPT world, this meant that you shouldn't be able to build two portfolios out of public securities such that one consistently outperformed the other. In the IGT world, there's no clear meaning to "fair price." The equivalent hypothesis is that capital is allocated to securities properly.

In MPT/EMH, if some good news comes out about a security, investors will buy it until its price goes up to the correct new value. The dynamics of an IGT/EMH world are different. If good news comes out, investors in a security will want to hold more of it. They will accomplish this not by buying more, but by the price going up and increasing the value of their original holdings. The buying pressure to make this happen has to come from the issuer, either buying back securities or, what amounts to the same thing, paying dividends. If the issuer happens to need more capital instead of less, it could instead raise new capital in a secondary offering, with new investors coming in to set the higher price.

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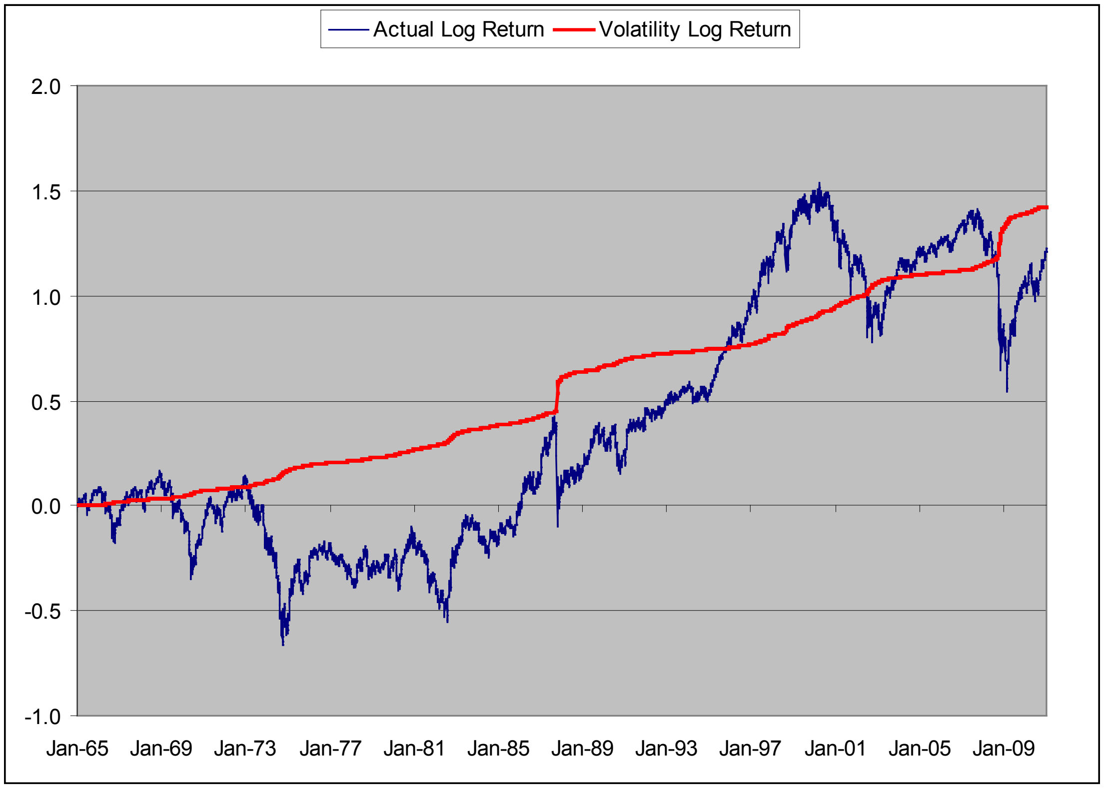
We can't say which one is a more accurate picture of the world because, as Eugene Fama famously wrote, any test of market efficiency is a joint test of market efficiency and market equilibrium. In order to test if markets are efficient, we need a model of what the correct equilibrium price is.

In the real world in which MPT was the dominant theory, the dominant model of market equilibrium was the Capital Asset Pricing Model (CAPM). It held that the expected excess return of any asset ("excess" means the return above either the risk-free rate of interest or above a zero Beta asset) is equal to the asset's Beta times the expected excess return of the market. This follows from MPT and EMH with some specific assumptions about the market and investors.

In the parallel universe of IGT we get a different formula. Here are both equations with μ representing expected return, ß the regression Beta on the market portfolio, σ the standard deviation of return, ω the proportion of the value of the security to the value of the market, and subscripts *s* for a security, *m* for the market, and 0 for the risk-free or zero-Beta asset.

MPT: $\mu_s - \mu_0 = ß_{s,m}\,(\mu_m - \mu_s)$
IGT: $\mu_s - \mu_0 = 2ß_{s,m}\,(\mu_m - \mu_0) + \omega_s \sigma_s^2$

Figure 1: S&P500 historical returns and IGT/CAPM prediction



The IGT CAPM results from simple assumptions and algebra. Public investors hold all their wealth in public securities, for that to be efficient under IGT $\mu_m - \mu_0$ has to be equal to σ_m^2. This is an approximation that we would assume is exactly true for the model. If we add or subtract an asset to or from the market, we need this relation to be unchanged. That implies as we add new assets to the market, the rate of return on existing assets goes up, because we have more demand chasing a fixed supply of capital. Similarly if we remove an asset from the market, the rate of return on remaining assets goes down.

$$(\mu_m - \mu_0) + \omega_s(\mu_s - \mu_0) = \sigma_m^2 + \omega_s^2\sigma_s^2 + 2\omega_s\rho_{s,m}\sigma_m\sigma_s$$
$$\omega_s(\mu_s - \mu_0) = \omega_s^2\sigma_s^2 + 2\omega_s\rho_{s,m}\sigma_m\sigma_s$$
$$\mu_s - \mu_0 = \omega_s\sigma_s^2 + 2\rho_{s,m}\sigma_m\sigma_s$$
$$\mu_s - \mu_0 = \omega_s\sigma_s^2 + 2\beta_{s,m}\sigma_m^2$$
$$\mu_s - \mu_0 = \omega_s\sigma_s^2 + 2\beta_{s,m}(\mu_m - \mu_0)$$

What immediately strikes you as odd about this equation is it doesn't seem to apply to the market as a whole. The Beta of the market is 1, so we get:

$$\mu_m - \mu_0 = \omega_m\sigma_m^2 + 2(\mu_m - \mu_0)$$

However, this is resolved when you realize ω_m is negative 1 and σ_m^2 equals $\mu_m - \mu_0$. If you remove the market from the market, the equation holds. It is only the example of the CAPM that leads you to expect that the average Beta asset (which must have a Beta of 1) has to have the average excess expected return (which has to equal $\mu_m - \mu_0$).

Nevertheless, the equation does not describe an equilibrium. It implies that assets are constantly moving in and out of the public markets: IPOs, secondary offerings, dividends, distributions, buy-backs and companies taken private.

The MPT/CAPM gathered tremendous empirical support during the 1960s and 70s, with work by Eugene Fama and others. The basic test for the IGT/CAPM would be to chart the return of the market and the return of the market squared. IGT/CAPM says these should be equal in the long run. We expect the actual returns to vary around the volatility returns, as the actual result can differ from the expected. But the deviations should not be too large or persistent. Figure 1 using the S&P 500 is roughly consistent with that assertion, although someone looking only at data from 1975 to 2000 would disagree, and that is a long period to explain as random error. *Unlike the MPT/CAPM, there is no equity risk premium puzzle in IGT/CAPM.*

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I have no idea whether more rigorous testing of IGT/CAPM would show it to be valid. It is an entirely different theory from MPT/CAPM. It cannot predict the price of a security, only the relation of a security's return and volatility to its supply. It does not suggest that all public investors should hold the same portfolio, nor does it suggest that more diversification is always better. In some ways it seems to be a better description of the world, especially the world before changes initiated by MPT arguments. Of course it has unrealistic assumptions, but I think it captures important aspects of the truth.